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A general solution for one-dimensional multistream heat exchangers and their networks

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Abstract

A mathematical model for predicting the steady-state thermal performance of one-dimensional (cocurrent and countercurrent) multistream heat exchangers and their networks is developed and is solved analytically for constant physical properties of streams. By introducing three matching matrices, the general solution can be applied to various types of one-dimensional multistream heat exchangers such as shell-and-tube heat exchangers, plate heat exchangers and plate–fin heat exchangers as well as their networks. The general solution is applied to the calculation and design of multistream heat exchangers. Examples are given to illustrate the procedures in detail. Based on this solution the superstructure model is developed for synthesis of heat exchanger networks. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Heat exchangers; Heat recovery; Optimization

1. Introduction

Multistream heat exchangers are widely used in process industries such as gas processing and petrochemical industries to exchange heat energy among more than two fluids with different supply temperatures because of their higher efficiency, more compact structure and lower costs than two-stream heat exchanger networks. A multistream plate–fin heat exchanger can even handle up to 10 process streams in a single unit [1]. The use of multistream heat exchangers is more cost-effective and can offer significant advantages over conventional two-stream heat exchangers in certain applications, especially in cryogenic plants [2–4]. However, the investigation on synthesis of heat exchanger networks using multistream heat exchangers is still limited because of lack of suitable calculation methods for the thermal performance of general multistream heat exchangers.

The multistream heat exchangers can be classified into two categories. One is multichannel heat exchanger in which there is no thermal interconnection between the walls separating the fluids, such as shell-and-tube heat exchangers and plate heat exchangers. The other is multistream plate–fin heat exchanger. The mathematical model and its analytical solution for the thermal performance of one-dimensional multistream plate–fin heat exchangers was first proposed by Kao [5]. Haseler [6] defined a bypass efficiency which describes heat transfer between non-adjacent layers in a plate–fin heat exchanger to illustrate the bypass effect. For multichannel heat exchangers a general solution of the temperature distributions was proposed by Wolf [7]. Many significant discussions on the general solution have been made [8–12]. Based on the pioneering research work of Kao [5] and Wolf [7], the thermal design problems of multistream plate–fin heat exchangers were solved by Luo et al. [13]. By introducing three matching matrices Roetzel and Luo proposed a general form of the analytical solution for various types of one-dimensional multistream heat exchangers and their networks [14]. In the present paper, their method is further developed and applied to the thermal calculation and design problems

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Nomenclature

$\mathbf{A}$	coefficient matrix of the governing equation system
A_f	total cross-sectional area of fins perpendicular to the fin height coordinate, m^2
Bi	Biot number of fins, $Bi = (h-\delta)\alpha_f F_f / A_f \lambda_f$, dimensionless
F	heat transfer area, m^2
$\mathbf{G}$	interchannel matching matrix
$\mathbf{G}'$	entrance matching matrix
$\mathbf{G}''$	exit matching matrix
h	fin height, m
k	overall heat transfer coefficient, W/m^2 K
L	length of the heat exchanger, m
M	number of channels
m	number of sections in a plate–fin heat exchanger
N	number of streams
n	number of layers in a block of a plate–fin heat exchanger
R	number of heat exchangers in a network
s	fin space, m
$\mathbf{T}$	fluid temperature vector, K
t	fluid temperature, K
U	heat transfer parameter, $U = kF/L$, $U_{f,ij} = \alpha_{f,ij}F_{f,ij}/L_j$, $U_{p,ij} = \alpha_{p,ij}F_{p,ij}/L_j$, W/m K
$\mathbf{U}$	matrix of eigenvectors of the governing equation system
W	width of the heat exchanger, m
$\dot{W}$	thermal flow rate, W/K
x	spatial coordinate along the length of the heat exchanger, m

Greek symbols

α	heat transfer coefficient, W/m^2 K
δ	fin thickness, m
η	fin efficiency
$\mathbf{\Lambda}$	vector of eigenvalues of the governing equation system
λ	heat conductivity, W/m K; also eigenvalues of the governing equation system
μ	fin bypass efficiency

Superscripts

$'$	entrance
$''$	exit

Subscripts

f	fin
p	plate
s	supply
t	target

of multistream heat exchangers and their networks. Examples are given to illustrate the procedures.

2. General mathematical model and its solution

Consider a generalized N-stream heat exchanger which consists of a bundle of M parallel channels ($M \geqslant N$). The fluid flowing through a channel exchanges heat with the fluids in all other channels. It is assumed: (1) The longitudinal heat conduction in the solid wall can be neglected. (2) There is no heat loss to the environment. (3) The heat transfer coefficients and the properties of the fluids and wall materials can be considered constant within each channel. The general mathematical model can be written as

$$\dot{W}_i \frac{\mathrm{d}t_i}{\mathrm{d}x} = \sum_{j=1}^{M} U_{ij}\left(t_j - t_i\right) \quad (i = 1,\ldots,M) \tag{1}$$

with $U_{ij} = U_{ji}$ and $U_{ii} = 0$.

It is convenient to rewrite Eq. (1) into a matrix form

$$\frac{\mathrm{d}\mathbf{T}}{\mathrm{d}x} = \mathbf{AT}, \tag{2}$$

where $\mathbf{A}$ is an $M \times M$ matrix

$$\mathbf{A} = \begin{bmatrix} -\frac{1}{\dot{W}_1}\sum_{l=1}^{M} U_{1l} & \frac{U_{12}}{\dot{W}_1} & \cdots & \frac{U_{1M}}{\dot{W}_1} \\ \frac{U_{21}}{\dot{W}_2} & -\frac{1}{\dot{W}_2}\sum_{l=1}^{M} U_{2l} & \cdots & \frac{U_{2M}}{\dot{W}_2} \\ \vdots & & \ddots & \\ \frac{U_{M1}}{\dot{W}_M} & \frac{U_{M2}}{\dot{W}_M} & \cdots & -\frac{1}{\dot{W}_M}\sum_{l=1}^{M} U_{Ml} \end{bmatrix}. \tag{3}$$

The positive value of $\dot{W}_i$ indicates that the fluid flows in the positive direction of the spatial coordinate and vice versa. If $\dot{W}_i$ and U_{ij} are constant in each channel (they may vary from channel to channel), the above ordinary differential equation system is linear and can be solved analytically. According to the theory of linear algebra the general solution of Eq. (2) is obtained in the matrix form as

$$\mathbf{T} = \mathbf{U}\mathrm{e}^{\Lambda x}\mathbf{D} \tag{4}$$

in which $\mathrm{e}^{\Lambda x} = \mathrm{diag}\{\mathrm{e}^{\lambda_i x}\}$ is a diagonal matrix and λ_i $(i = 1,\ldots,M)$ are the eigenvalues of matrix $\mathbf{A}$. $\mathbf{U}$ is an $M \times M$ square matrix whose columns are the eigenvectors of the corresponding eigenvalues. Eq. (4) is valid only if the eigenvalues differ from each other. It has been proved that all eigenvalues of matrix $\mathbf{A}$ are real, however, Eq. (4) might have multiple eigenvalues [9–11].

A practical method to avoid multiple eigenvalues is to add very small random deviations to the input parameters such as $\dot{W}_i$ or U_{ij}. Such small deviations have almost no effect on the results.

The coefficient vector **D** in Eq. (4) is determined by the boundary conditions. To get a general expression of the boundary conditions, we introduce the following three matching matrices:

Interchannel matching matrix **G**. It is an $M \times M$ matrix whose elements g_{ij} are defined as the ratio of the thermal flow rate flowing from channel j into channel i to that flowing through channel i.

Entrance matching matrix $\mathbf{G}'$. It is an $M \times N$ matrix whose elements g'_{ik} are defined as the ratio of the thermal flow rate flowing from the entrance of stream k to channel i to that flowing through channel i.

Exit matching matrix $\mathbf{G}''$. It is an $N \times M$ matrix whose elements g''_{ki} are defined as the ratio of the thermal flow rate flowing from channel i to the exit of stream k to that flowing out of the exit of stream k.

From energy balance at the boundaries, i.e., the entrances of M channels, we have

$$\mathbf{T}(\mathbf{x}') = \mathbf{G}'\mathbf{T}' + \mathbf{G}\mathbf{T}(\mathbf{x}'') \tag{5}$$

in which

$$\mathbf{T}(\mathbf{x}') = \left[t_1(x'_1), t_2(x'_2), \ldots, t_M(x'_M)\right]^{\mathrm{T}}, \tag{6}$$

$$\mathbf{T}(\mathbf{x}'') = \left[t_1(x''_1), t_2(x''_2), \ldots, t_M(x''_M)\right]^{\mathrm{T}}. \tag{7}$$

$\mathbf{x}'$ and $\mathbf{x}''$ are the coordinate vectors of the entrances and exits of M channels, respectively.

Substitution of the boundary conditions, Eq. (5), into Eq. (4) yields

$$\mathbf{T} = \mathbf{U}\mathrm{e}^{\Lambda x}\left(\mathbf{V}' - \mathbf{G}\mathbf{V}''\right)^{-1}\mathbf{G}'\mathbf{T}', \tag{8}$$

where $\mathbf{V}'$ and $\mathbf{V}''$ are two $M \times M$ matrices, whose elements are given as

$$v'_{ij} = u_{ij}\mathrm{e}^{\lambda_j x'_i}, \tag{9}$$

$$v''_{ij} = u_{ij}\mathrm{e}^{\lambda_j x''_i}, \tag{10}$$

respectively. The outlet fluid temperatures of the exchanger can then be expressed explicitly as

$$\mathbf{T}'' = \mathbf{G}''V''\left(\mathbf{V}' - \mathbf{G}\mathbf{V}''\right)^{-1}\mathbf{G}'\mathbf{T}'. \tag{11}$$

Eq. (11) is general for one-dimensional heat exchangers. The input data are the heat transfer parameters and thermal flow rates given in **A**, the flow arrangement set by **G**, $\mathbf{G}'$ and $\mathbf{G}''$ and the coordinates given in $\mathbf{x}'$ and $\mathbf{x}''$. The coefficient matrix **A** also depends on the type of the exchanger to be considered.

3. Applications of the general solution

To use the general solution one should at first divide the exchanger into several sections according to the construction of the exchanger. Each section contains several channels. The fluids flow through the channels and exchange heat with the fluids in other channels. The sections should be divided such that there are no entrances or exits of streams inside the sections and the fluid properties in each channel can be considered constant. After the channel configuration has been made, it is easy to get the matching matrices **G**, $\mathbf{G}'$ and $\mathbf{G}''$.

The major task to use the general solution is the calculation of the coefficient matrix **A**. For multichannel heat exchangers Eq. (3) can be used directly to calculate the coefficient matrix **A**. A lot of elements of **A** become zero because there is no heat exchange between corresponding channels. However, the mathematical model of temperature distribution in a multistream plate–fin heat exchanger, which also contains energy equations of separating plates and fins, differs from Eq. (1). By eliminating the temperatures of separating plate and fins in the energy equation of fluids, Luo et al. [13] transformed the governing equation system into the form of Eq. (2). The corresponding coefficient matrix **A** should be specially calculated.

In the following examples it will be illustrated in detail how to determine the matrices **A**, **G**, $\mathbf{G}'$ and $\mathbf{G}''$. The examples also show how to use the general solution to solve the design problems of multistream heat exchangers.

3.1. Shell-and-tube heat exchangers

In a multistream shell-and-tube heat exchanger each tube-side fluid exchanges heat only with the shell-side fluid. There is no direct thermal contact between any two tube-side fluids. The input matrices of an N-stream E-type exchanger (one shell pass, arbitrary number of tube passes, no split) have been given in [14]. Here a more complicated example will be discussed.

3.1.1. Example 1

In this example, a three-stream E-type shell-and-tube heat exchanger is used to heat two cold streams with one hot stream. The data taken from [2] are presented in Table 1. The exchanger is divided into three sections and seven channels, as shown in Fig. 1, which yields

Table 1
Problem data for Example 1

Stream		T_s (K)	T_t (K)	$\dot{W}$ (W/K)
1	H1	420	370	−8000
2	C1	300	350	4000
3	C2	280	320	5000

$k = 1.1$ kW/m^2 K for all matches

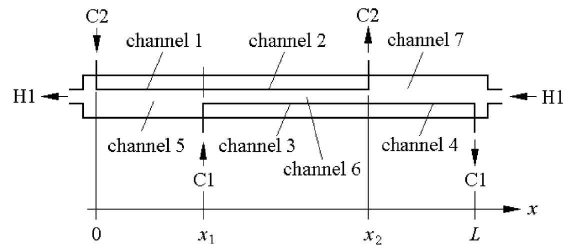

Fig. 1. Construction of the three-stream shell-and-tube heat exchanger.

$$\mathbf{A} = \begin{bmatrix} -\frac{U_{H1C2}}{\dot{W}_{C2}} & 0 & 0 & 0 & \frac{U_{H1C2}}{\dot{W}_{C2}} & 0 & 0 \\ 0 & -\frac{U_{H1C2}}{\dot{W}_{C2}} & 0 & 0 & 0 & \frac{U_{H1C2}}{\dot{W}_{C2}} & 0 \\ 0 & 0 & -\frac{U_{H1C1}}{\dot{W}_{C1}} & 0 & 0 & \frac{U_{H1C1}}{\dot{W}_{C1}} & 0 \\ 0 & 0 & 0 & -\frac{U_{H1C1}}{\dot{W}_{C1}} & 0 & 0 & \frac{U_{H1C1}}{\dot{W}_{C1}} \\ \frac{U_{H1C2}}{\dot{W}_{H1}} & 0 & 0 & 0 & -\frac{U_{H1C2}}{\dot{W}_{H1}} & 0 & 0 \\ 0 & \frac{U_{H1C2}}{\dot{W}_{H1}} & \frac{U_{H1C1}}{\dot{W}_{H1}} & 0 & 0 & -\frac{U_{H1C2}+U_{H1C1}}{\dot{W}_{H1}} & 0 \\ 0 & 0 & 0 & \frac{U_{H1C1}}{\dot{W}_{H1}} & 0 & 0 & -\frac{U_{H1C1}}{\dot{W}_{H1}} \end{bmatrix}, \tag{12}$$

$$\mathbf{G} = \begin{bmatrix} 0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0 \\ 0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0 \end{bmatrix}, \quad \mathbf{G}' = \begin{bmatrix} 0&0&1 \\ 0&0&0 \\ 0&1&0 \\ 0&0&0 \\ 0&0&0 \\ 0&0&0 \\ 1&0&0 \end{bmatrix}, \quad \mathbf{G}'' = \begin{bmatrix} 0&0&0&0&1&0&0 \\ 0&0&0&1&0&0&0 \\ 0&1&0&0&0&0&0 \end{bmatrix}, \tag{13}$$

$$\mathbf{x}' = [0, x_1, x_1, x_2, x_1, x_2, L]^{\mathrm{T}}, \quad \mathbf{x}'' = [x_1, x_2, x_2, L, 0, x_1, x_2]^{\mathrm{T}}.$$

By setting $L = 1$, for given values of variables x_1, x_2, U_{H1C1} and U_{H1C2}, the outlet stream temperatures can be calculated. The problem given in Table 1 for minimum heat transfer area becomes

$$\begin{aligned} \min \quad & (L - x_1)U_{H1C1}/k + x_2 U_{H1C2}/k \\ \text{s.t.} \quad & t_{t,C1} - t_2'' = 0, \quad t_{t,C2} - t_3'' = 0, \quad x_1 - x_2 \leqslant 0; \\ & 0 \leqslant U_{H1C1}, \quad 0 \leqslant U_{H1C2}, \quad 0 \leqslant x_1 \leqslant L, \quad 0 \leqslant x_2 \leqslant L. \end{aligned} \tag{14}$$

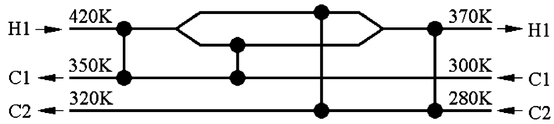

Fig. 2. Equivalent two-stream heat exchanger network.

The results are $x_1 = 0.2703$, $x_2 = 0.5483$, $U_{H1C1} = 3.438$ kW/m K and $U_{H1C2} = 4.294$ kW/m K, which gives the minimum heat transfer area of 4.42 m^2. This exchanger is equivalent to the two-stream heat exchanger network shown in Fig. 2.

3.2. Plate heat exchangers

A plate heat exchanger consists of a number of parallel channels formed by a stack of heat transfer plates. According to the combination of the plates with holes or blanks located at the four corners of the plate and the additional manifold axes if necessary, various flow patterns may be created in a multistream plate heat exchanger, which can be classified into three categories: series flow pattern, parallel flow pattern and complex flow pattern. It is assumed that in the plate heat exchanger the fluid in each channel has thermal contact only with the two adjacent channels. The corresponding coefficient matrix of the governing equation system reads

$$\mathbf{A} = \begin{bmatrix} -\frac{U_{12}}{\dot{W}_1} & \frac{U_{12}}{\dot{W}_1} & 0 & \cdots & 0 \\ \frac{U_{21}}{\dot{W}_2} & -\frac{U_{21}+U_{23}}{\dot{W}_2} & \frac{U_{23}}{\dot{W}_2} & 0 & 0 \\ 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & \frac{U_{M-1,M-2}}{\dot{W}_{M-1}} & -\frac{U_{M-1,M-2}+U_{M-1,M}}{\dot{W}_{M-1}} & \frac{U_{M-1,M}}{\dot{W}_{M-1}} \\ 0 & \cdots & 0 & \frac{U_{M,M-1}}{\dot{W}_M} & -\frac{U_{M,M-1}}{\dot{W}_M} \end{bmatrix}, \tag{15}$$

where M is the number of channels.

3.2.1. Example 2

As an example, a three-stream plate heat exchanger with countercurrent parallel arrangement shown in Fig. 3 is taken for the analysis. The data presented in Table 1 are used again. The numbers of channels for C1 and C2 are M_{C1} and M_{C2}, respectively. Thus, $M_{H1} = M_{C1} + M_{C2} + 1$, $M = M_{H1} + M_{C1} + M_{C2}$. Since the values of k_{H1C1} and k_{H1C2} given in Table 1 are constant, $k_{H1C1} = k_{H1C2} = k$, we have $U = kF_p/L$ for all plates in which F_p is the effective heat transfer area of one plate.

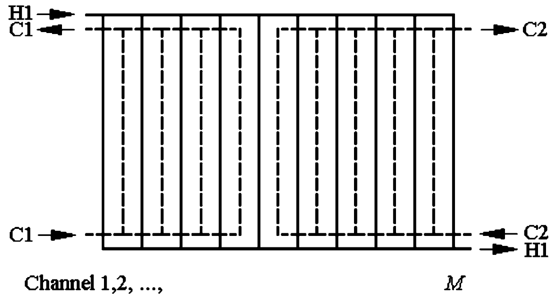

Fig. 3. Three-stream plate heat exchanger with countercurrent parallel arrangement.

From Fig. 3 we have,

$$x_i' = \begin{cases} L, & i \text{ is odd}, \\ 0, & i \text{ is even}, \end{cases} \quad x_i'' = \begin{cases} 0, & i \text{ is odd}, \\ L, & i \text{ is even}. \end{cases} \tag{16}$$

It is further assumed that the thermal flow rates are uniformly distributed in their channels. Thus, the thermal flow rates in each channel are given as

$$\dot{W}_i = \begin{cases} \dot{W}_{H1}/M_{H1}, & i \text{ is odd}, \\ \dot{W}_{C1}/M_{C1}, & i \text{ is even and } i \leqslant 2M_{C1}, \\ \dot{W}_{C2}/M_{C2}, & i \text{ is even and } i > 2M_{C1}. \end{cases} \tag{17}$$

According to the channel connection shown in Fig. 3, we also have $\mathbf{G} = \mathbf{0}$. The non-zero elements of $\mathbf{G}'$ and $\mathbf{G}''$ are given by

$$g_{ik}' = 1 \quad \text{if} \begin{cases} & k = 1, \ i \text{ is odd}, \\ \text{or} & k = 2, \ i \text{ is even and } i \leqslant 2M_{C1}, \\ \text{or} & k = 3, \ i \text{ is even and } i > 2M_{C1}, \end{cases}$$

$$g_{ki}'' = \begin{cases} 1/m_{H1}, & k = 1 \text{ and } i \text{ is odd}, \\ 1/m_{C1}, & k = 2, \ i \text{ is even and } i \leqslant 2M_{C1}, \\ 1/m_{C2}, & k = 3, \ i \text{ is even and } i > 2M_{C1}. \end{cases} \tag{18}$$

By setting $L = 1$, for given values of integer variables M_{C1} and M_{C2}, the outlet stream temperatures can be calculated. The design problem given in Table 1 becomes

$$\begin{aligned} \min \quad & M_{C1} + M_{C2} \\ \text{s.t.} \quad & t_{t,C1} - t_2'' \leqslant 0, \quad t_{t,C1} - t_3'' \leqslant 0; \\ & 0 < M_{C1}, \quad 0 < M_{C2}. \end{aligned} \tag{19}$$

The results are show in Table 2 for $F_p = 0.2$ and 0.1 m^2, respectively.

Table 2
Results of Example 2

Stream		$F_p = 0.2$ m^2		$F_p = 0.1$ m^2	
		T'' (K)	M	T'' (K)	M
1	H1	365.4	14	367.9	25
2	C1	352.5	7	350.6	13
3	C2	325.3	6	322.9	11
F (m^2)		5.2 m^2		4.8 m^2	

3.3. Plate–fin heat exchangers

A plate–fin heat exchanger consists of fins separated by flat plates, clamped and brazed together, as shown in Fig. 4. The plates separating two fluids function as the primary heat transfer surface. The fin sheets between the adjacent plates hold the plates together and form a secondary surface for heat transfer. The space of fin sheets between two plates forms a flow channel and is known as a layer. A multistream plate–fin heat exchanger contains more than two streams flowing through different layers and sections of the exchanger. The exchanger usually consists of many passage blocks which are repetitively arranged. Each block consists of n layers. Since there is a very large number of layers in an exchanger, we usually assume that the behaviour of a block can adequately describe that of the entire exchanger, therefore only n layers need to be analysed. There are two kinds of block arrangements. One is sequential arrangement and the other is symmetrical arrangement. For the sequential arrangement of the blocks, e.g.,

$$\cdots \underbrace{\text{A B C D}}_{\text{Block } j-1} \underbrace{\text{A B C D}}_{\text{Block } j} \underbrace{\text{A B C D}}_{\text{Block } j+1} \cdots,$$

the layer number $i = n + 1$ points to the first layer in the upper block ($i = 1$); the layer number $i = 0$ points to the nth layer in the lower block ($i = n$). For the symmetrical arrangement, e.g.,

$$\cdots \underbrace{\text{D C B A}}_{\text{Block } j-1} \underbrace{\text{A B C D}}_{\text{Block } j} \underbrace{\text{D C B A}}_{\text{Block } j+1} \cdots,$$

the layer number $i = n + 1$ points to the same layer in the upper block ($i = n$); the layer number $i = 0$ points to the same layer in the lower block ($i = 1$). The symmetrical arrangement also means that the block is thermally insulated at the upper and lower surfaces. If the whole exchanger is analysed, the symmetrical arrangement should be adopted.

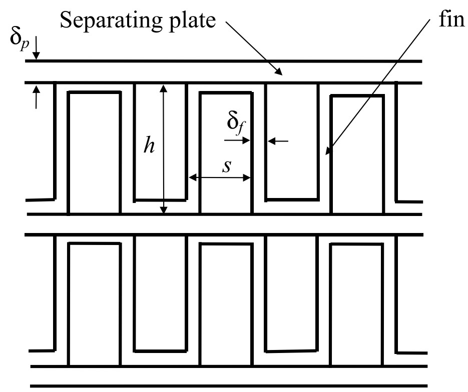

Fig. 4. Configuration of the plate–fin heat exchanger surface.

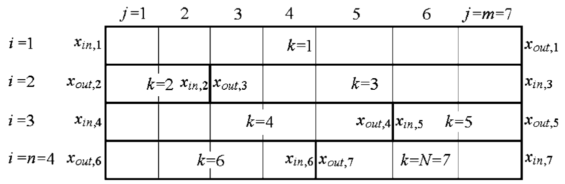

Fig. 5. Arrangement of the streams, layers and sections in a plate–fin heat exchanger.

Consider a block in a multistream plate–fin heat exchanger, which has n layers and is divided along the exchanger length into m sections according to the inlet and outlet positions of the streams as shown in Fig. 5. Therefore the whole exchanger consists of mn channels. The elements of the $mn \times mn$ coefficient matrix $\mathbf{A}$ are given by Luo et al. [13] for both sequential and symmetrical block arrangements as

$$a_{(i-1)m+j,(i-1)m+j} = -\frac{U_{\mathrm{p},ij}+\eta_{ij}U_{\mathrm{f},ij}}{\dot{W}_{ij}} \times \left[1-\frac{1}{2}\left(p_{(i-1)m+j,(i-1)m+j} + p_{[I(i+1)-1]m+j,(i-1)m+j}\right)\right], \tag{20a}$$

$$a_{(i-1)m+j,l} = \frac{U_{\mathrm{p},ij}+\eta_{ij}U_{\mathrm{f},ij}}{2\dot{W}_{ij}}\left(p_{(i-1)m+j,l} + p_{[I(i+1)-1]m+j,l}\right)$$
$$(l = 1,\ldots,mn;\ l \neq (i-1)m+j) \tag{20b}$$

for $i = 1,\ldots,n$ and $j = 1,\ldots,m$, in which

$$I(i) = \begin{cases} 1, & i = n+1 \text{ and sequential arrangement,} \\ i, & \text{others} \end{cases} \tag{21}$$

and the fin efficiency η_{ij} is defined as

$$\eta_{ij} = \tanh\left(\sqrt{Bi_{ij}}/2\right)/\left(\sqrt{Bi_{ij}}/2\right) \tag{22}$$

with

$$Bi_{ij} = \left(h_{ij}-\delta_{ij}\right)\alpha_{\mathrm{f},ij}F_{\mathrm{f},ij}/A_{\mathrm{f},ij}\lambda_{\mathrm{f},ij}. \tag{23}$$

To calculate the plate temperatures the fin bypass efficiency introduced by Haseler [6] is used which is defined as

$$\mu_{ij} = \frac{2}{\sqrt{Bi_{ij}}\sin\mathrm{h}\sqrt{Bi_{ij}}}. \tag{24}$$

$\mathbf{P}$ is the coefficient matrix of plate temperatures,

$$\mathbf{T}_{\mathrm{p}} = \mathbf{PT}. \tag{25}$$

For sequential block arrangement

$$\mathbf{P} = \mathbf{Q}^{-1}\mathbf{C}, \tag{26}$$

where $\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{C}$ are $mn \times mn$ matrices. The non-zero elements of $\mathbf{Q}$ and $\mathbf{C}$ are given as follows:

$i = 1,\ j = 1,\ldots,m$:

$$q_{j,(n-1)m+j} = -\mu_{nj}U_{\mathrm{f},nj}, \tag{27a}$$

$$q_{jj} = U_{\mathrm{p},1,j} + \left(\eta_{1,j}+\mu_{1,j}\right)U_{\mathrm{f},1,j} + U_{\mathrm{p},nj} + \left(\eta_{nj}+\mu_{nj}\right)U_{\mathrm{f},nj}, \tag{27b}$$

$$q_{j,m+j} = -\mu_{1,j}U_{\mathrm{f},1,j}, \tag{27c}$$

$$c_{j,(n-1)m+j} = U_{\mathrm{p},nj} + \eta_{nj}U_{\mathrm{f},nj}, \tag{27d}$$

$$c_{jj} = U_{\mathrm{p},1,j} + \eta_{1,j}U_{\mathrm{f},1,j}. \tag{27e}$$

$i = n,\ j = 1,\ldots,m$:

$$q_{(n-1)m+j,(n-2)m+j} = -\mu_{n-1,j}U_{\mathrm{f},n-1,j}, \tag{28a}$$

$$q_{(n-1)m+j,(n-1)m+j} = U_{\mathrm{p},nj} + \left(\eta_{nj}+\mu_{nj}\right)U_{\mathrm{f},nj} + U_{\mathrm{p},n-1,j} + \left(\eta_{n-1,j}+\mu_{n-1,j}\right)U_{\mathrm{f},n-1,j}, \tag{28b}$$

$$q_{(n-1)m+j,j} = -\mu_{nj}U_{\mathrm{f},nj}, \tag{28c}$$

$$c_{(n-1)m+j,(n-2)m+j} = U_{\mathrm{p},n-1,j} + \eta_{n-1,j}U_{\mathrm{f},n-1,j}, \tag{28d}$$

$$c_{(n-1)m+j,(n-1)m+j} = U_{\mathrm{p},nj} + \eta_{nj}U_{\mathrm{f},nj}. \tag{28e}$$

$i = 2,\ldots,n-1,\ j = 1,\ldots,m$:

$$q_{(i-1)m+j,(i-2)m+j} = -\mu_{i-1,j}U_{\mathrm{f},i-1,j}, \tag{29a}$$

$$q_{(i-1)m+j,(i-1)m+j} = U_{\mathrm{p},ij} + \left(\eta_{ij}+\mu_{ij}\right)U_{\mathrm{f},ij} + U_{\mathrm{p},i-1,j} + \left(\eta_{i-1,j}+\mu_{i-1,j}\right)U_{\mathrm{f},i-1,j}, \tag{29b}$$

$$q_{(i-1)m+j,im+j} = -\mu_{ij}U_{\mathrm{f},ij}, \tag{29c}$$

$$c_{(i-1)m+j,(i-2)m+j} = U_{\mathrm{p},i-1,j} + \eta_{i-1,j}U_{\mathrm{f},i-1,j}, \tag{29d}$$

$$c_{(i-1)m+j,(i-1)m+j} = U_{\mathrm{p},ij} + \eta_{ij}U_{\mathrm{f},ij}. \tag{29e}$$

For symmetrical block arrangement $\mathbf{P}$ is an $m(n+1) \times mn$ matrix

$$p_{l,(i-1)m+j} = \begin{cases} p^*_{l,(i-1)m+j}, & i = 1,\ldots,n-1 \\ p^*_{l,(n-1)m+j} + p^*_{l,nm+j}, & i = n \end{cases}$$
$$(l = 1,\ldots,m(n+1);\ j = 1,\ldots,m), \tag{30}$$

where

$$\mathbf{P}^* = \mathbf{Q}^{-1}\mathbf{C} \tag{31}$$

and $\mathbf{Q}$ and $\mathbf{C}$ are $m(n+1) \times m(n+1)$ matrices whose non-zero elements for $1 < i \leqslant n$ are the same as Eqs. (29a)–(29e). For the first and last plates we have, $i = 1,\ j = 1,\ldots,m$:

$$q_{jj} = U_{\mathrm{p},1,j} + \left(\eta_{1,j}+\mu_{1,j}\right)U_{\mathrm{f},1,j}, \tag{32a}$$

$$q_{j,m+j} = -\mu_{1,j}U_{\mathrm{f},1,j}, \tag{32b}$$

Table 3
Comparison of predicted outlet fluid temperatures with the experimental data of a four-stream plate–fin heat exchanger

Stream	$\dot{W}$ (kW/K)	α (kW/m^2 K)	T_{in} (°C)	$T_{out,exp.}$ (°C)	$T_{out,cal.}$ (°C)
A	1.354	1.644	41.93	32.43	32.53
B	−0.9604	1.791	34.93	39.40	39.03
C	−0.5902	1.465	31.06	39.62	39.39
D	−0.8015	0.8189	21.98	27.23	26.82

$$c_{jj} = U_{p,1,j} + \eta_{1,j} U_{f,1,j}. \quad (32c)$$

$i = n+1, \; j = 1, \ldots, m$:

$$q_{nm+j,(n-1)m+j} = -\mu_{nj} U_{f,nj}, \quad (33a)$$

$$q_{nm+j,nm+j} = U_{p,nj} + \left(\eta_{nj} + \mu_{nj}\right) U_{f,nj}, \quad (33b)$$

$$c_{nm+j,(n-1)m+j} = U_{p,nj} + \eta_{nj} U_{f,nj}. \quad (33c)$$

The matrices $\mathbf{G}$, $\mathbf{G}'$ and $\mathbf{G}''$ and the vectors $\mathbf{x}'$ and $\mathbf{x}''$ should be set according to the particular configuration of the exchanger.

3.3.1. Example 3

Take a four-stream aluminium plate–fin heat exchanger as an example, of which the experimental data were given by Li et al. [15]. The exchanger is used to cool the product stream A and heat the product stream D to given temperatures. The arrangement of the exchanger is B A C/D A B A C/D A B A C/D A B A C/D A B. However, only one block B A C/D A in sequential arrangement is taken for the calculation. The channel arrangement is shown in Fig. 6. In the exchanger the hot water stream A is cooled by the cold water streams B, C and D. Offset strip fins ($h = 4.7$ mm, $s = 2.0$ mm, $\delta = 0.3$ mm) are used for channels A, B and C, and perforated rectangular fins ($h = 4.7$ mm, $s = 4.2$ mm, $\delta = 0.6$ mm) for channel D. The parameters for the ith layer and jth section can be calculated by

$$U_{f,ij} = 2\alpha_{ij}(h_{ij} - \delta_{ij})W/s_{ij}, \quad U_{p,ij} = 2\alpha_{ij}(s_{ij} - \delta_{ij})W/s_{ij},$$
$$Bi_{ij} = 2\alpha_{ij}(h_{ij} - \delta_{ij})^2/(\lambda_f \delta_{ij}),$$

where W is the width of the exchanger, $W = 130$ mm. The heat conductivity of fins $\lambda_f = 191.58$ W/m K. The heat transfer coefficients and thermal flow rates are

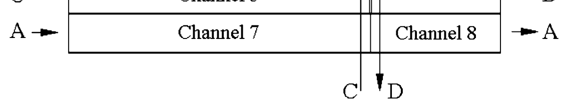

Fig. 6. Construction of the four-stream plate–fin heat exchanger.

given in Table 3. Thus, the coefficient matrix A can be obtained. According to Fig. 6, the coordinate vectors and matching matrices are given as follows:

$$\mathbf{x}' = [0.925, 1.24, 0, 0.925, 0.925, 1.24, 0, 0.925]^T \text{ (m)},$$

$$\mathbf{x}'' = [0, 0.925, 0.925, 1.24, 0, 0.925, 0.925, 1.24]^T \text{ (m)}.$$

The non-zero elements of $\mathbf{G}$, $\mathbf{G}'$ and $\mathbf{G}''$ are

$$g_{12} = g_{43} = g_{87} = 1, \quad g'_{22} = g'_{31} = g'_{53} = g'_{64} = g'_{71} = 1,$$
$$g''_{14} = g''_{18} = 0.5, \quad g''_{21} = g''_{35} = g''_{46} = 1.$$

Table 3 also gives the comparison between the measured outlet fluid temperatures and the calculated ones. A good agreement is achieved between them.

3.4. Heat exchanger networks

The general solution can also be applied to the networks of two-stream heat exchangers and one-dimensional multistream heat exchangers by considering the network as a general multistream heat exchanger. However, if the network contains a large number of exchangers, the coefficient matrix of the governing equation system would be enlarged, which might cause difficulties in calculating its eigenvalues and eigenvectors.

Let us consider a network with N streams and R heat exchangers. From Eq. (11) we have already obtained the temperature coefficient matrices of R individual exchangers

$$\mathbf{V}_r = \mathbf{G}''_r \mathbf{V}'_r \left(\mathbf{V}'_r - \mathbf{G}_r \mathbf{V}''_r\right)^{-1} \mathbf{G}'_r \quad (r = 1, 2, \ldots, R). \quad (34)$$

We assume that each stream in an exchanger occupies one channel. Therefore, the network consists of M channels ($M = \sum_{l=1}^{R} N_l$) and the channel number of the nth stream in the rth exchanger can be set as $m = n + \sum_{l=1}^{r-1} N_l$ where N_l is the number of streams in the lth exchanger. Thus, according to the energy balance at the entrance of each channel, the outlet stream temperature of the network can be expressed as

$$\mathbf{T}'' = \mathbf{G}''\mathbf{V}(\mathbf{I} - \mathbf{G}\mathbf{V})^{-1}\mathbf{G}'\mathbf{T}' \quad (35)$$

in which

Table 4
Problem data for Example 4

Stream		T_s (°C)	T_t (°C)	$\dot{W}$ (kW/K)	α (kW/m^2 K)
1	H1	150	60	−20	0.05
2	H2	90	60	−80	0.4
3	HU	181	180	1075	1
4	C1	20	125	25	0.1
5	C2	25	100	30	0.6
6	CU	10	15	80	0.6

$$\mathbf{V} = \begin{bmatrix} \mathbf{V}_1 & & & \mathbf{0} \\ & \mathbf{V}_2 & & \\ & & \ddots & \\ \mathbf{0} & & & \mathbf{V}_R \end{bmatrix}. \tag{36}$$

3.4.1. Example 4

The example given in Table 4 is taken from [16]. Its temperature–enthalpy diagram is shown in Fig. 7. According to Fig. 7, we first consider a network consisting of five multichannel heat exchangers and one two-stream heat exchanger, as shown in Fig. 8. The streams in each exchanger are arranged as follows:

EX1: H1 C3 H2, symmetric,
EX2: H1 C1 H2, symmetric,
EX3: H1 C1 H2 C2, sequential,
EX4: C1 H1 C2, symmetric,
EX5: C1 H3 C2, symmetric,
EX6: H1 C1, sequential.

The heat transfer parameters are calculated by

$$U_{i-j} = \frac{F_{i-j}}{L\left(1/\alpha_i + 1/\alpha_j\right)}, \tag{37}$$

where F_{i-j} is the heat transfer area between streams i and j. The design problem is to find F_{i-j} for all matches so that the sum of them reaches minimum

$$\begin{aligned} \min \quad & \sum F_{i-j} \\ \text{s.t.} \quad & t''_{\mathrm{H}k} - t_{\mathrm{t,H}k} = 0 \ \ (k = 1,2,3), \\ & t_{\mathrm{t,C}k} - t''_{\mathrm{C}k} \geqslant 0 \ \ (k = 1,2); \quad 0 \leqslant F_{i-j}. \end{aligned} \tag{38}$$

To calculate the outlet stream temperatures, the necessary matrices of each exchanger are given as follows, respectively, in which the length of exchangers is set to be $L = 1$.

$$\mathbf{A}_1 = \begin{bmatrix} -\frac{U_{1-3}}{\dot{W}_{\mathrm{H1}}} & 0 & \frac{U_{1-3}}{\dot{W}_{\mathrm{H1}}} \\ 0 & -\frac{U_{2-3}}{\dot{W}_{\mathrm{H2}}} & \frac{U_{2-3}}{\dot{W}_{\mathrm{H2}}} \\ \frac{U_{1-3}}{\dot{W}_{\mathrm{CU}}} & \frac{U_{2-3}}{\dot{W}_{\mathrm{CU}}} & -\frac{U_{1-3}+U_{2-3}}{\dot{W}_{\mathrm{CU}}} \end{bmatrix},$$

$$\mathbf{A}_2 = \begin{bmatrix} -\frac{U_{4-6}}{\dot{W}_{\mathrm{H1}}} & 0 & \frac{U_{4-6}}{\dot{W}_{\mathrm{H1}}} \\ 0 & -\frac{U_{5-6}}{\dot{W}_{\mathrm{H2}}} & \frac{U_{5-6}}{\dot{W}_{\mathrm{H2}}} \\ \frac{U_{4-6}}{\dot{W}_{\mathrm{C1}}} & \frac{U_{5-6}}{\dot{W}_{\mathrm{C1}}} & -\frac{U_{4-6}+U_{5-6}}{\dot{W}_{\mathrm{C1}}} \end{bmatrix},$$

$$\mathbf{A}_3 = \begin{bmatrix} -\frac{U_{7-9}+U_{7-10}}{\dot{W}_{\mathrm{H1}}} & 0 & \frac{U_{7-9}}{\dot{W}_{\mathrm{H1}}} & \frac{U_{7-10}}{\dot{W}_{\mathrm{H1}}} \\ 0 & -\frac{U_{8-9}+U_{8-10}}{\dot{W}_{\mathrm{H2}}} & \frac{U_{8-9}}{\dot{W}_{\mathrm{H2}}} & \frac{U_{8-10}}{\dot{W}_{\mathrm{H2}}} \\ \frac{U_{7-9}}{\dot{W}_{\mathrm{C1}}} & \frac{U_{8-9}}{\dot{W}_{\mathrm{C1}}} & -\frac{U_{7-9}+U_{8-9}}{\dot{W}_{\mathrm{C1}}} & 0 \\ \frac{U_{7-10}}{\dot{W}_{\mathrm{C2}}} & \frac{U_{8-10}}{\dot{W}_{\mathrm{C2}}} & 0 & -\frac{U_{7-10}+U_{8-10}}{\dot{W}_{\mathrm{C2}}} \end{bmatrix},$$

$$\mathbf{A}_4 = \begin{bmatrix} -\frac{U_{11-12}+U_{11-13}}{\dot{W}_{\mathrm{H1}}} & \frac{U_{11-12}}{\dot{W}_{\mathrm{H1}}} & \frac{U_{11-13}}{\dot{W}_{\mathrm{H1}}} \\ \frac{U_{11-12}}{\dot{W}_{\mathrm{C1}}} & -\frac{U_{11-12}}{\dot{W}_{\mathrm{C1}}} & 0 \\ \frac{U_{11-13}}{\dot{W}_{\mathrm{C2}}} & 0 & -\frac{U_{11-13}}{\dot{W}_{\mathrm{C2}}} \end{bmatrix},$$

$$\mathbf{A}_5 = \begin{bmatrix} -\frac{U_{14-15}+U_{14-16}}{\dot{W}_{\mathrm{HU}}} & \frac{U_{14-15}}{\dot{W}_{\mathrm{HU}}} & \frac{U_{14-16}}{\dot{W}_{\mathrm{HU}}} \\ \frac{U_{14-15}}{\dot{W}_{\mathrm{C1}}} & -\frac{U_{14-15}}{\dot{W}_{\mathrm{C1}}} & 0 \\ \frac{U_{14-16}}{\dot{W}_{\mathrm{C2}}} & 0 & -\frac{U_{14-16}}{\dot{W}_{\mathrm{C2}}} \end{bmatrix},$$

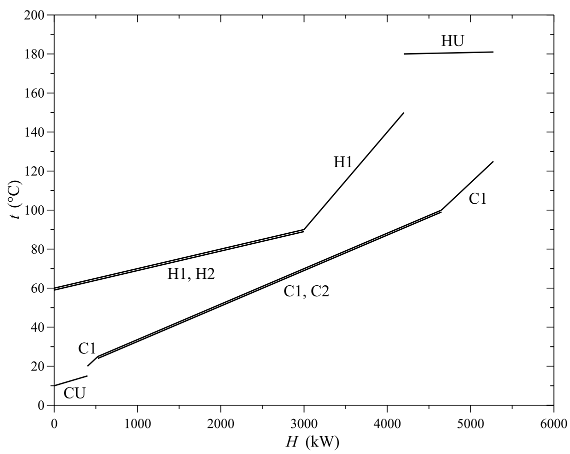

Fig. 7. Temperature–enthalpy diagram of Example 3.

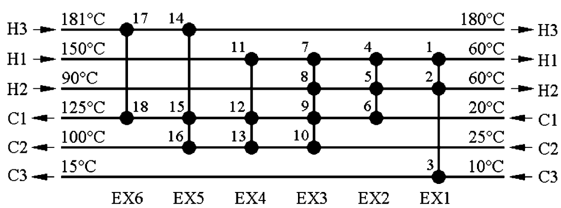

Fig. 8. Heat exchanger network of Example 4.

$$\mathbf{A}_6 = \begin{bmatrix} -\frac{U_{17\text{-}18}}{\dot{W}_{HU}} & \frac{U_{17\text{-}18}}{\dot{W}_{HU}} \\ \frac{U_{17\text{-}18}}{\dot{W}_{C1}} & -\frac{U_{17\text{-}18}}{\dot{W}_{C1}} \end{bmatrix},$$

$$\mathbf{x}'_1 = \mathbf{x}'_2 = [1,1,0]^T, \quad \mathbf{x}'_3 = [1,1,0,0]^T,$$

$$\mathbf{x}'_4 = \mathbf{x}'_5 = [1,0,0]^T, \quad \mathbf{x}'_6 = [1,0]^T,$$

$$\mathbf{x}''_1 = \mathbf{x}''_2 = [0,0,1]^T, \quad \mathbf{x}''_3 = [0,0,1,1]^T,$$

$$\mathbf{x}''_4 = \mathbf{x}''_5 = [0,1,1]^T, \quad \mathbf{x}''_6 = [0,1]^T,$$

$$\mathbf{G}_i = \mathbf{0}, \quad \mathbf{G}'_i = \mathbf{G}''_i = \mathbf{I} \quad (i = 1,2,\ldots,6).$$

The non-zero elements of the matching matrices of the whole network are given as,

$$g_{1,4} = g_{2,5} = g_{4,7} = g_{5,8} = g_{7,11} = g_{9,6} = g_{12,9} = g_{13,10} = g_{14,17} = g_{15,12} = g_{16,13} = g_{18,15} = 1,$$

$$g'_{3,6} = g'_{6,4} = g'_{8,2} = g'_{10,5} = g'_{11,1} = g'_{17,3} = 1,$$

$$g''_{1,1} = g''_{2,2} = g''_{3,14} = g''_{4,18} = g''_{5,16} = g''_{6,3} = 1.$$

Solving Eq. (38) we obtain,

EX1: $F_{H1\text{–}C3} = 128.96\ m^2$, $F_{H2\text{–}C3} = 6.02\ m^2$,
EX2: $F_{H1\text{–}C1} = 45.58\ m^2$, $F_{H2\text{–}C1} = 0\ m^2$,
EX3: $F_{H1\text{–}C1} = 346.53\ m^2$, $F_{H1\text{–}C2} = 0\ m^2$,
$F_{H2\text{–}C1} = 269.27\ m^2$, $F_{H2\text{–}C2} = 384.10\ m^2$,
EX4: $F_{H1\text{–}C1} = 282.48\ m^2$, $F_{H1\text{–}C2} = 218.01\ m^2$,
EX5: $F_{H3\text{–}C1} = 37.81\ m^2$, $F_{H3\text{–}C2} = 5.08\ m^2$,
EX6: $F_{H3\text{–}C1} = 101.48\ m^2$.

The total heat transfer area is 1825.32 m^2. Since $F_{H2\text{–}C1}$ in EX2 and $F_{H1\text{–}C2}$ in EX3 are zero, EX2 reduces to a two-stream heat exchanger and EX3 should be symmetrically arranged.

This network is equivalent to the network consisting of 11 two-stream heat exchangers. According to the cost equation [16]:

$$\text{cost} = 8.6 + 0.67\ \text{Area}^{0.83} \quad (\text{area in } m^2,\ \text{cost in k\$}) \tag{39}$$

the cost of exchangers is 578.76 k$. This value is much smaller than that given by Briones and Kokossis [16]. For the same problem, the application of their model yielded a network consisting of five exchangers. They did not give the structure of the network. In their example, the matches of design A are given as: H1–C1, H1–C3, H2–C1, H2–C2, H3–C2. The total heat transfer area is 3314.1 m^2 and the cost of the exchangers is 699.2 k$. It should be pointed out that in this problem there are five outlet stream temperatures to be targeted (the sixth one is determined by the energy balance of the whole network). Therefore the degree of freedom is equal to the number of variable parameters minus five. If only five exchangers are used, the heat transfer area of each exchanger is fixed for given structure of the network. The possible structure of their design A is shown in Fig. 9. The heat transfer area of each exchange can be obtained as: $F_{H1\text{–}C3} = 151.59\ m^2$, $F_{H2\text{–}C1} = 162.19\ m^2$, $F_{H2\text{–}C2} = 519.30\ m^2$, $F_{H1\text{–}C1} = 2462.94\ m^2$, $F_{H3\text{–}C2} = 29.44\ m^2$. The total area and total cost of exchangers are 3325.46 m^2 and 700.80 k$, respectively, which are close to the results given in [16].

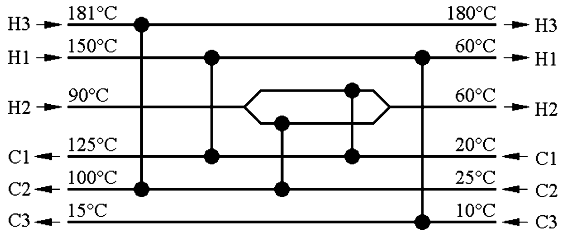

Fig. 9. Heat exchanger network of Example 4 according to [16].

Now we consider the minimum total cost of heat exchangers as the object function. Starting from a general multichannel heat exchanger network illustrated in Fig. 10, synthesis of the two-stream heat exchanger network by using the present general solution for Example 4 with minimum total cost of heat exchangers yields a network with six exchangers shown in Fig. 11, whose areas are $F_{H1\text{–}C3} = 151.59\ m^2$, $F_{H1\text{–}C1} = 552.34\ m^2$, $F_{H2\text{–}C1} = 344.11\ m^2$, $F_{H2\text{–}C2} = 425.87\ m^2$, $F_{H1\text{–}C2} = 238.80\ m^2$ and $F_{H3\text{–}C1} = 157.56\ m^2$, respectively. The total heat transfer area is 1870.27 m^2 and total cost of exchangers is 516.46 k$.

The synthesis method used here is based on a stage-wise superstructure [2] and the whole temperature field

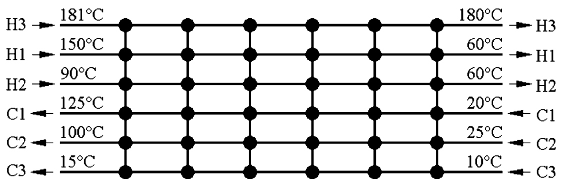

Fig. 10. Start structure of the multichannel heat exchanger network of Example 4 with minimum cost of heat exchangers.

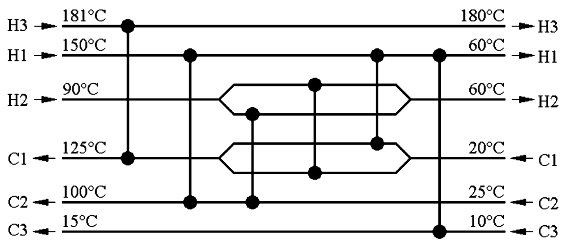

Fig. 11. Two-stream heat exchanger network of Example 4 with minimum cost of heat exchangers.

of the network calculated with the general solution. The advantage using this method is obvious because the synthesis problem reduces into a general non-linear optimization task with outlet temperature constraints and other additional constraints if needed. The binary variables determining whether the corresponding exchanger exists are not necessary because usually the value of n in the cost equation

$$\text{cost} = a + bF^n \tag{40}$$

is less than one. It is convenient to use

$$\text{cost} = \begin{cases} a + bF^n, & F \geqslant F_{\min}, \\ \left(a + bF_{\min}^n\right)F/F_{\min}, & F < F_{\min}. \end{cases} \tag{41}$$

In this case the optimization will automatically yield both the area and cost of an unnecessary exchanger to zero.

4. Conclusions

A general form of the analytical solution for various types of one-dimensional multistream heat exchangers and their networks is proposed. The methods for the use of the general solution are illustrated in detail for one-dimensional flow shell-and-tube heat exchangers, plate heat exchangers, plate–fin heat exchangers and heat exchanger networks. The solution is valid for any types of two-stream heat exchangers by introducing the correction factor of logarithmic mean temperature difference.

The outlet temperatures of the streams in a multistream heat exchanger or heat exchanger network are explicitly given by Eq. (11) or Eq. (35). Therefore it is easy to obtain the outlet stream temperatures for an existing heat exchanger or heat exchanger network. However, for design problems of multistream heat exchanger and their networks no simple relationship for the calculation of heat transfer area is available. Therefore a constrained optimization algorithm is needed to determine unknown heat transfer areas and other parameters under a set of constraints, as shown in Examples 1, 2 and 4.

The general solution is also applied to the synthesis of heat exchanger networks. Based on this solution the stage-wise superstructure method is developed to solve the synthesis problem of two-stream as well as multistream heat exchanger networks. For Example 4 a better network than that of [16] is obtained.

The present solution can be applied to the case of variable physical properties by dividing the flow passages into several channels and assuming that in each channel the fluid properties are constant. Iterations are needed because the fluid properties should be calculated according to the mean temperatures of the fluids in the corresponding channels.

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